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Research Article

Stochastic Modelling of Daily Rainfall for Decision Making in Water Management in Benin (West Africa)

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Abstract

The pattern and amount of rainfall are among the most important factors that affect directly or indirectly all sectors depending on water availability (such as agriculture, water supply, hydroelectricity production, etc.). Knowledge of dry and wet spell characteristics of rainfall plays an important role in the management of water resources. The objective of this paper is to apply stochastic process for describing and analyzing the daily rainfall pattern in Benin. To this end, this study used first and second-order Markov chain to analyse the wet and dry spells and then used one-parameter exponential and two-parameter gamma distributions to produce wet day rainfall amount. The results of rainfall occurrence revealed that the probabilities of having two successive dry days and the probabilities of having three successive dry days are highest among all other transitions probabilities. Moreover, analysis of the characteristics of dry and wet spells duration reveals that dry spell duration fluctuates around the mean more than the wet spell duration. Regarding daily rainfall generation, the use of the mean absolutely relative error performance criteria allows us to conclude that the two-parameter gamma distribution is consistently better than the one-parameter exponential distribution at simulating rainfall in the study area at daily, monthly and yearly scale.

Keywords

Introduction

Research studies on the pattern and amount of rainfall are very important for the assessment of hydrological consequences, such as flooding and droughts in the context of climate change. Prediction of rainfall has remained an unsolved problem till now [1]. In fact, the future probability of occurrence of rainfall can be used for crop planning and management, as well as water management decisions, as the risk due to weather uncertainty can be reduced [2].

Markov chain models are very common for stochastic rainfall generation. A typical Markov chain rainfall model is composed of two parts: a rainfall occurrence model that uses a transition probability between wet and dry days, and a rainfall magnitude model that uses a probability distribution of wet day rainfall depths (commonly a gamma or exponential distribution) fitted to the observed data [2]. Several authors have used Markov chains to model the daily occurrence of rainfall, dry and wet spells duration and generation

of rainfall amount. After the work of Gabriel KR and Neumann J [3] who applied the Markov chain model successfully to describe Tel Aviv daily rainfall data, a number of researchers have applied a similar technique to study rainfall in widely different geographical regions. However, except for a few early studies Afouda A [4-6], not much work has been carried out to model the wet and dry spell sequences of daily rainfall observed in Benin. The probability of a transition from a dry to a wet day or from a wet to a dry day is of fundamental importance. Markov chains are used to represent these discrete events. Markov chains specify the state of each day as wet or dry and develop a relation between the state of the current day and the states of the preceding days. The order of the Markov chain is the number of preceding days taken into account. Most Markov chain models referred in the literature are first order [7]. Afouda A [4-6] showed the Markovian character of the wet and dry sequences distribution over a year in Benin. Jovanovic C et al., [8] used a Markov chain model for the simulation of wet and dry spell sequences. They found that this procedure for simulating daily rainfall over a vast catchment area has given satisfactory results concerning the seasonal rainfall depths. Selvaraj RS and Selvi ST [2] used a Markov chain for describing and analyzing the rainfall pattern at Aduthurai. They suggested that first-order Markov chain with two parameters gamma distributions is adequate to generate daily rainfall sequences at Aduthurai. Srikanthan R and McMahon TA [7] developed a Markov chain model, which uses a two-state Markov chain process with two parameters (wet-to-wet and dry-to-dry transition probabilities) to simulate rainfall occurrence and a gamma distribution with two parameters (mean and standard deviation of wet day rainfall) to simulate wet day rainfall depths. He also found that this model is suitable to simulate the rainfall pattern. Wet and dry spells are among the most significant natural phenomenon because they affect the water resources, crop yield and food demand, causing surpluses and deficits within a region. They constitute two main physical characteristics of rainfall occurrence and rainfall amount. The disasters induced by the alternation of wet and dry spells have increased due to climate change effects, with their temporal and spatial coverage during the last two decades because of anthropogenic and naturally unexpected developments, such as the global warming, implying greenhouse effect and rainfall deficiency [9]. A deeper analysis of wet and dry spell sequences using Markov chain is still lacking in Benin. Therefore, the objective of this study is to use Markov chain model to analyse the wet and dry spells and to use also exponential and gamma distributions to produce wet day rainfall amount. Wet and dry durations are helpful in quantitative description of drought and flood and flashs flood occurrence assessments respectively.

Materials and Methods

Characteristics of the study area and data used

Benin is located on the Guinea Coast of West Africa (between 6° 25' and 12° 30' North latitude and 0° 45' and 4° East longitude) and is bordered to the west by Togo, to the east by Nigeria, and to the north by Niger and Burkina Faso (Figure 1). It stretches 670 kilometers from the Bight of Benin in the south to the Niger River in the north and has a coastline stretching 122 km from east to west
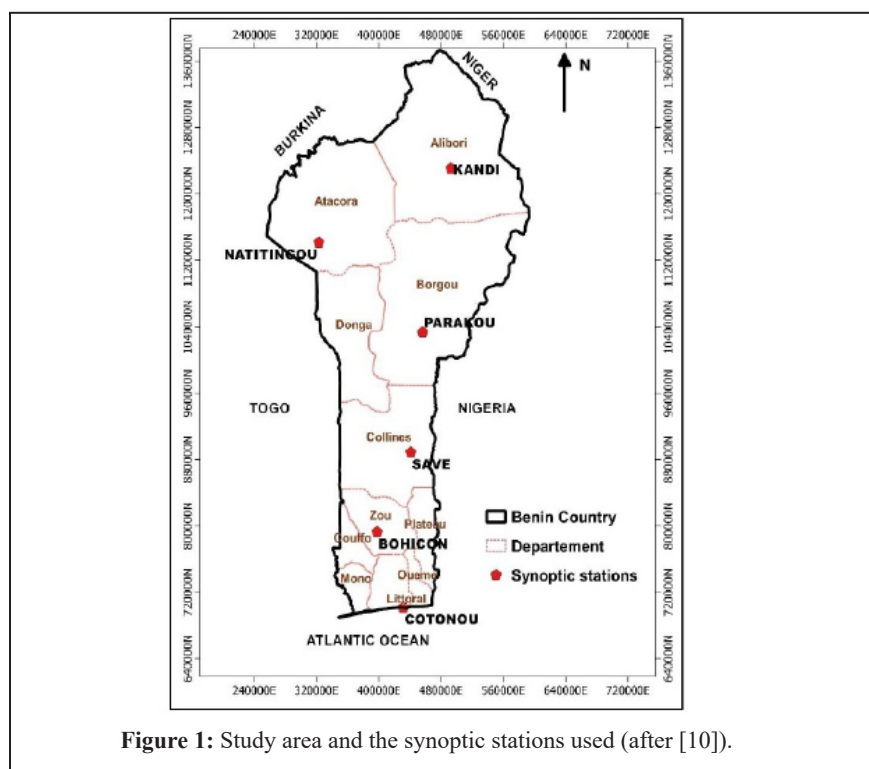

Figure 1: Study area and the synoptic stations used (after [10]).

Benin has a sub-humid tropical climate that is largely controlled by the West African monsoon circulation. The bulk of the annual precipitation is received during the rainy season of the boreal summer [11]. The annual mean rainfall is around 1200 mm in the northern Benin, whereas in the the southern part, it is around 1300 mm. The dry season is characterized by dry, dusty, northeasterly Harmattan winds. The temperature varies from 20° to 35° in the study area. A large variety of soils can be found in Benin. Fersialitic soils with high gravel contents are dominant on the crystalline basement, while ferralitic and hydromorphic soils prevail in the sedimentary basins in the south of Benin [12]. In general, small scale variability of the soils is very high, and many soil types are associated with other soil types. The geomorphology of Benin is closely linked to its geologic structure. The main geologic units of Benin are the re-metamorphised Precambrian basement complex of the Dahomeyen Series and three sedimentary basins, located in North (Kandi basin and Precambrian Volta basin) and South Benin (Coastal basin).

Meteorological data (i.e., daily rainfall data) used in this study come from the six synoptic stations of Benin and were provided by the Benin Meteorological Department, ASCENA (Agency for Air Navigation Safety in Africa and Madagascar). The rainfall data are considered for the rainy season over the period 1952-2011. Table 1 gives the geographic description of the synoptic stations.

Stations	Longitude (°C)	Latitude (°C)	Elevation (m)
Cotonou	2.38	6.35	4
Bohicon	2.07	7.17	166
Savee	2.47	8.03	198
Parakou	2.6	9.35	392
Natitingou	1.38	10.32	460
Kandi	2.93	11.13	290
Table 1: Geographical descriptions of the synoptic stations.			

Methodology

Markov chain modelling: Rainfall exhibits a strong variability in time and space. Hence, its stochastic modelling is not an easy task [13]. Chain-dependent models treat the occurrence and intensity of daily rainfall events separately [14-18]. The term "chain dependence" reflects the statistical structure of the occurrence sequence. For instance, for a first-order Markov chain process, the chain dependence means that the state at time t only depends on the state at time t - 1, and is independent of states at other times. If the present state depends on more than one previous state, the time sequence is said to follow a higher-order chain-dependent process and the number of related previous states is termed the chain order.

The daily rainfall occurrence series is generally assumed to follow a chain-dependent process in stochastic weather models. The occurrence of rainfall can be defined as $X_t = 1$, if it is wet

on day t, and is $X_t = 0$, otherwise.

Once the model order is determined, the occurrence process can be completely characterized by the transition probabilities.

The first-order Markov process is the simplest and most widely used. The occurrence process of a two-state first-order chain series is completely described with the function

$$P_{ij} = \Pr(X_t = j \mid X_{t-1} = i, X_{t-2} = i_{t-1},..., X_1 = i_1) = \Pr(X_t = j \mid X_{t-1} = i) \quad (1)$$

where $i, j, i_1,..., i_{t-1} = 0,1$.

Here, Pr is the probability and P_{ij} are the transition probabilities to change between state i (dry and wet) and state j (dry and wet).

The probability of rainfall, on a given day, is based on the wet or dry status of the previous day, which can be defined in terms of two transition probabilities, P_{01} and P_{11}:

$P_{01} = \Pr$ {rainfall on day t|no rainfall on day t-1}   (2)

$P_{11} = \Pr$ {rainfall on day t | rainfall on day t-1}   (3)

Since precipitation either occurs or does not occur on a given day, the two complementary transition probabilities are $P_{00} = 1 - P_{01}$ and $P_{10} = 1 - P_{11}$. The corresponding transition probability matrix can be written in the form

$$P = \begin{pmatrix} P_{00} & P_{01} \\ P_{01} & P_{11} \end{pmatrix} \quad (4)$$

A generalization of the first-order Markov model is to consider higher-order Markov model such as the second-order model. eq. (1) can be extended to the second-order Markov chains following eq. (5):

$$P_{ijk} = \Pr(X_t = k \mid X_{t-1} = j, X_{t-2} = i, X_{t-3} = i_{t-3},..., X_1 = i_1)$$

$$= \Pr(X_t = k \mid X_{t-1} = j, X_{t-2} = i) \quad (5)$$

where $i, j, k, i_1,..., i_{t-1} = 0,1$. P_{ijk} are the transition probabilities with constraints

$$P_{ij0} + P_{ij1} = 1, i, j = 0.1. \quad (6)$$

The corresponding transition probability matrix can be written in the form:

$$P = \begin{pmatrix} P_{000} & P_{001} \\ P_{010} & P_{011} \\ P_{110} & P_{111} \\ P_{100} & P_{101} \end{pmatrix} \quad (7)$$

The number of parameters required to characterize rainfall occurrence increases exponentially with the order of Markov

process. This means that two and four parameters must be estimated for first and second-order Markov models, respectively.

Although the first-order two-state chain-dependent (Markov chain) model is most commonly used for studying daily rainfall, it often underestimates variance and extremes. In an effort to eliminate these differences, studies suggest that extending the model's complexities can greatly improve the approximating and reproducing capability of the model [19-23]. As such, in this study, we used first and second-order Markov models for describing rainfall occurrence at six synoptic stations in Benin.

Characteristics of dry and wet spells duration: It is possible to define any season of the year by a sequence of successive spells, wet and dry. Wet and dry spells can be quantified based on basic characteristics, such as duration of wet and dry spells, defined as a period of d consecutive rainy (non-rainy) days. In the present study, a wet spell is defined as a continuous run of a limited number of days in each of which at least 0.1mm of daily rainfall had been recorded. Any day credited with less than 0.1mm or no rain is considered a dry day.

Let us describe the variable that describes the duration of dry spell in the form:

$$S_0 = \min\{n; X_t = 0\} \qquad (8)$$

where n is the number of times that a dry sequence has been observed. According to (4), the characteristics of dry spell duration, when using a Markov chain of order 1, can be summarized in the following equations:

- mathematical expectation (i.e., mean) of the dry spell duration:

$$m_1(S_0) = \frac{1}{1 - P_{00}} \qquad (9)$$

- Variance of the dry spell duration:

$$m_2(S_0) = \frac{1 + P_{00}}{(1 - P_{00})^2} \qquad (10)$$

- Standard deviation of the dry spell duration:

$$\sigma(S_0) = \sqrt{P_{00}/(1 - P_{00})} \qquad (11)$$

- Coefficient of variation of the dry spell duration:

$$C_v(S_0) = \sqrt{P_{00}} \qquad (12)$$

The same calculation can be performed for the characteristics of wet spell duration by replacing P_{00} by P_{11} in eq. (9), (10), (11) and (12).

Modelling wet and dry spells sequences: The probability of rainfall, on a given day, is based on the wet or dry status of the previous day, which can be defined in terms of two transition probabilities, P_{01} and P_{11}. In such case, the conditional probabilities given by eq. (2) and (3) could be written in the form of eq. (13) and (14):

$$P_{11} = \Pr\{\text{wet day / previous day wet}\} \qquad (13)$$

$$P_{01} = \Pr\{\text{wet day / previous day dry}\} \qquad (14)$$

The probability distribution of a dry spell of length m can be defined as m successive dry days followed by a wet day. Similarly, the probability of a wet spell of length k can be defined as k successive wet days followed by a dry day. These probabilities distributions were found to be geometric [24,25]. Thus, the probability of a wet spell with a length of k days and the probability of a dry spell with a length of m days are:

$$\Pr\{x = k\} = (1 - P_{11})P_{11}^{(k-1)} \qquad (15)$$

$$\Pr\{y = m\} = P_{01}(1 - P_{01})^{(m-1)} \qquad (16)$$

Using eq. (15) and (16), $\Pr\{x = k\}$ and $\Pr\{y = m\}$ will be computed for $k, m = 1,2,...,30$ and then summed up sequentially.

The cumulative distribution for wet sequences is $1 - P_{11}^k$, whereas the one for dry sequences is $1 - (1 - P_{01})^m$.

Stochastic model for generating daily rainfall amount: For a predicted rainy day, two probability distribution functions are available to produce the daily rainfall amount [26]. The first is the one-parameter exponential distribution, which has a probability density function given by

$$f(x) = \lambda e^{-\lambda x} \qquad (17)$$

where x is the daily rainfall intensity and λ is the distribution parameter, often called the rate parameter. The other function is the two-parameter gamma distribution. The probability density function for this distribution is given by

$$f(x) = \frac{(x/\beta)^{\alpha-1} \exp[-x/\beta]}{\beta \Gamma(\alpha)} \qquad (18)$$

where α and β are the two distribution parameters (α and β are respectively the shape and scale parameters), and $\Gamma(\alpha)$ indicates the gamma function evaluated at α. These methods are widely used to generate daily rainfall amount.

All model parameters are estimated from the observed dataset

by using Maximum Likelihood Estimator (MLE).

The two models will be compared to investigate which one produces daily rainfall time series that have statistical properties comparable to those of existing records in the investigated study area.

Results and Discussion

Rainfall occurrence

The results of the first and second-order Markov chain modelling are given in tables 2 and 3. It can be seen from these tables that, the probabilities of a dry spell are maximum in all cases. In other words, the transition probabilities P_{00} and P_{000} are highest among all other transitions probabilities.

north Benin. Meanwhile, a wet day will be followed with the highest probability by another wet day at Natitingou station. In fact, Natitingou area is known as the water tower of Benin.

Regarding the second-order Markov chain, the probabilities of having three successive dry days are much higher than those of having two successive dry days follows by a wet day, at all the studied stations. Also, the probabilities of having a dry day follows by a wet day and a dry day are greater than those of having a dry day follows by two successive wet days. Moreover, the probabilities of having a wet day follows by two successive dry days are greater than those of having a wet day follows by a dry day and a wet day. Finally, the probabilities of having two successive wet days' follows by a dry day are greater than those of having three successive wet days. The

Stations / Transition Probabilities	Cotonou	Bohicon	Save	Parakou	Kandi	Natitingou
P_{00}	0.8	0.79	0.8	0.81	0.81	0.82
P_{01}	0.2	0.21	0.2	0.19	0.19	0.18
P_{10}	0.55	0.59	0.55	0.55	0.59	0.52
P_{11}	0.45	0.41	0.45	0.45	0.41	0.48

Table 2: Transition probabilities for the first-order Markov chain.

Stations / Transition Probabilities	Cotonou	Bohicon	Save	Parakou	Kandi	Natitingou
P_{000}	0.77	0.76	0.76	0.76	0.76	0.79
P_{001}	0.23	0.24	0.24	0.24	0.24	0.21
P_{010}	0.62	0.68	0.64	0.67	0.66	0.66
P_{011}	0.38	0.32	0.36	0.33	0.34	0.34
P_{110}	0.74	0.74	0.73	0.74	0.72	0.74
P_{111}	0.26	0.26	0.27	0.26	0.28	0.26
P_{100}	0.67	0.69	0.71	0.71	0.7	0.71
P_{101}	0.33	0.31	0.29	0.29	0.3	0.29

Table 3: Transition probabilities for the second-order Markov chain.

Indeed, for the first-order Markov chain, the probabilities of having two successive dry days are much higher than those of having a dry day follows by a wet day, at all the studied stations. Likewise, the probabilities of having a wet day follows by a dry day are greater than those of having two successive wet days. The highest probability that a dry day will be followed by a dry day too is being registered at Kandi station, in the

highest probability of lack of rainfall in three consecutive days was found at Natitingou station, whereas two wet days will be followed with the highest probability by another wet day at Kandi station.

The implication of the above results is that there is likely to have drier days than wet days in rainy season. These

findings are consistent with those obtained by Afouda A and Adisso P [27], and Wagoussi OT [28]. The fact that dry periods are more persistent than wet periods leads to a shortage in river discharge and therefore in water resources. These results are worrying beacause it is well known that Natitingou area (i.e., mountain area) constitutes the water supply for the country.

Therefore, we can imagine the negative impact that an increase in dry spells lengths could have on the runoff and thus on activities linked to water availability over agro-pastoral areas. Such information is useful for water resources management, as it can improve the decision making process, which is aimed to optimize the availability of water for drinking and agricultural purposes. The succession of dry days can be attributed to a drop of rainfall intensities in the region and to a decrease in the number of wet days.

Figure 2 presents the variations of the first-order Markov

in P_{11}. These results are in line with those obtained by Jimoh OD and Webster P [29] who investigated the intra-annual variation of the Markov chain parameters for seven sites in Nigeria.

Analysis of the characteristics of dry and wet spells duration

Tables 4 and 5 give respectively the characteristics of dry and wet spell duration. It can be seen from these tables that the mean for dry spell duration varies from 4.7 to 5.5 days, whereas the mean for wet spell duration varies from 1.69 to 1.92 days. These results show that the mean for dry spell duration are greater that the mean for wet spell duration. Regarding the coefficient of variation, we can notice that the dry spell duration fluctuates around the mean more than the wet spell duration. Exploitation of such information can have potential benefits in the areas of decision making in water resources and agricultural management.
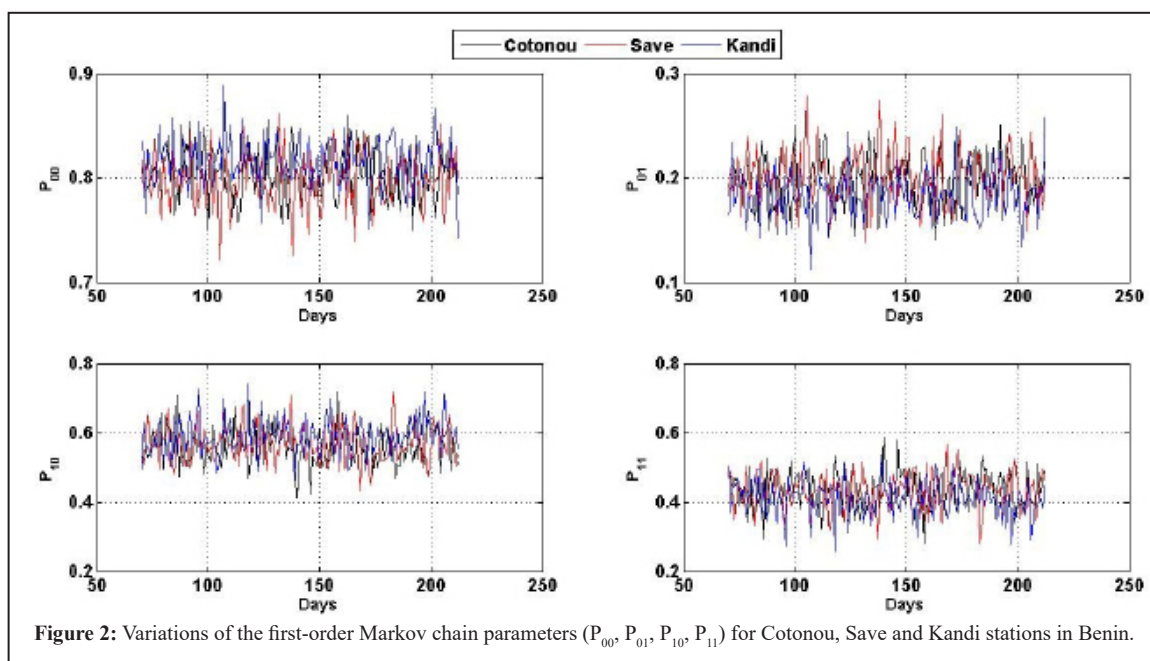

Figure 2: Variations of the first-order Markov chain parameters (P_{00}, P_{01}, P_{10}, P_{11}) for Cotonou, Save and Kandi stations in Benin.

chain parameters for some of the studied stations in Benin (Cotonou for the littoral zone in the south, Save for the transition zone in the center and Kandi for the semi arid Soudanian zone in the north) over the rainy season. Transition probabilities show considerable variation throughout the rainy season. There was a systematic variation in P_{01} (probability of a wet day following a dry day) as one moves northwards and a limited variation

Stations	Mean	Standard deviation σ	Coefficient of variation C_v
Cotonou	5	2	0.89
Bohicon	4.7	1.9	0.88
Savè	5	2	0.89
Parakou	5.2	2.1	0.9
Kandi	5.5	2.2	0.91
Natitingou	5.2	2.1	0.9
Table 4: Characteristics of dry spell duration (days).			

Stations	Mean	Standard deviation σ	Coefficient of variation C_v
Cotonou	1.82	0.90	0.67
Bohicon	1.69	0.83	0.64
Savè	1.82	0.90	0.67
Parakou	1.82	0.90	0.67
Kandi	1.69	0.83	0.64
Natitingou	1.92	0.96	0.69

Table 5: Characteristics of wet spell duration (days).

Moreover, wet and dry spell sequences have been modelled in the present study. Figure 3 presents the cumulative distribution function of wet spell duration and dry spell duration. From this figure, we can observe that the cumulative probability tends to 1 when the wet spell duration exceeds approximately 20 days, whereas the cumulative probability tends to 1 when the dry spell duration exceeds approximately 7 days. Knowledge of dry and wet spell occurrence could be very useful in water management in West Africa, especially in Benin country.

standard deviations of daily rainfall. This indicates that both distributions underestimate the high-frequency variability of rainfall. Likewise, both distributions underestimate the all-time maximum daily rainfall. This is understandable because neither the exponential nor the gamma distribution is tailed to generated extreme rainfall events. It is well-documented that extreme rainfall values follow different distribution functions [26]. Both distributions, however, perform relatively well in producing monthly and annual mean rainfall, while they underestimate the standard deviation of monthly rainfall. The standard deviation of annual rainfall is also considerably underestimated for both. This indicates also, as in the case of daily scale, that the exponential and gamma distributions underestimate the inter-annual and intra-annual variability of rainfall. However, the gamma distribution is consistently better than the exponential distribution at simulating rainfall in the study stations. Indeed, by using the Mean Absolutely Relative Error (MARE) we noticed that, for exponential distribution, this performance criteria varies from 0.31% to 0.38% for the mean rainfall, 0.28% to 0.35% for the standard deviation of rainfall and 0.41% to 0.48% for the maximum rainfall, whereas for the gamma distribution the MARE varies from
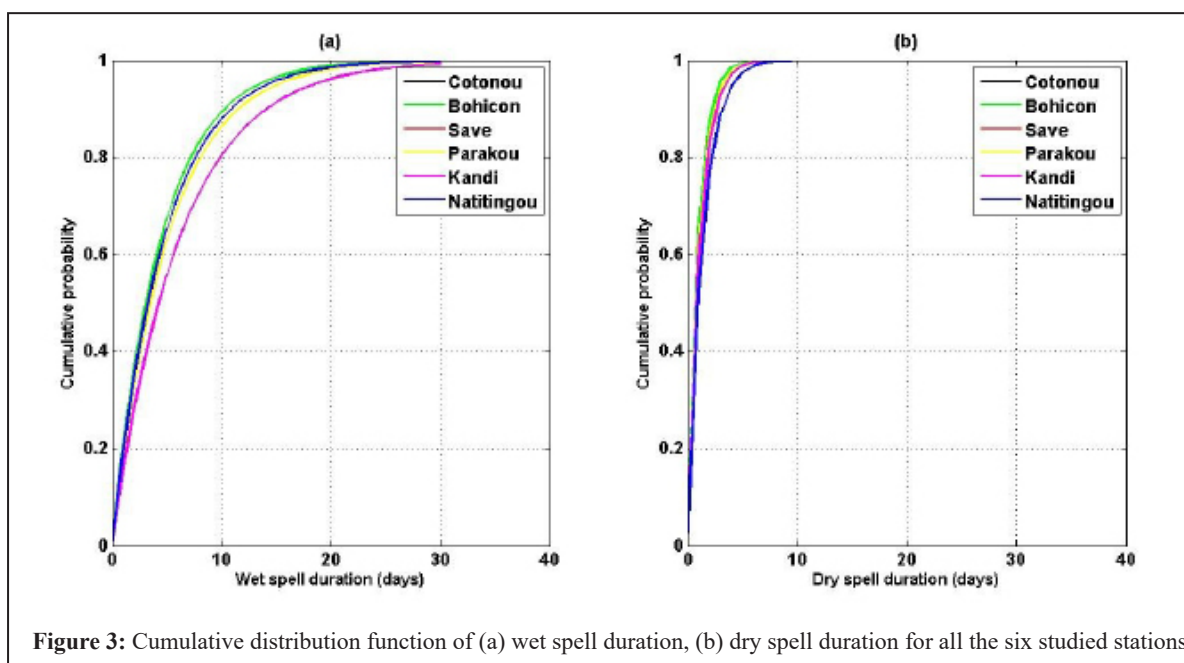
Figure 3: Cumulative distribution function of (a) wet spell duration, (b) dry spell duration for all the six studied stations

Generation of rainfall amount

The wet day precipitations are simulated with exponential and gamma distributions, respectively and the two generated datasets are compared in terms of accurately producing rainfall amount. The results show that both the exponential and gamma distributions reproduce the mean daily rainfall very well (Table 6). However, they both underestimate the

0.21% to 0.29% for the mean rainfall, 0.20% to 0.27% for the rainfall standard deviation and 0.30% to 0.40% for the maximum rainfall at daily, monthly and yearly scale over the six synoptic stations. Moreover, figure 4 shows the MARE, at different temporal scale, for the mean, standard deviation and maximum of rainfall for the two distribution models. These findings are in line with previous works [2,26] which also confirmed the superiority of gamma distribution over

Stations	Statistics	Daily			Monthly			Yearly		
		OBS	Exp	Gam	OBS	Exp	Gam	OBS	Exp	Gam
Cotonou	Mean	6.7	5.5	6.8	244.9	197	220	1327.5	1203.8	1253.3
	Stdev	18.2	11.3	15.4	143.6	91.3	95.8	362.2	174.9	207.9
	Max	129.2	84.7	105.5	434.7	225.8	395.8	2470.2	1580.5	1749.3
Bohicon	Mean	4.1	5.1	4.7	240.5	166.8	197.5	1295.9	1120.2	1198.6
	Stdev	11.2	14.6	9.5	143.6	39.7	83.3	343.6	144.3	280.7
	Max	99.6	80.1	98.2	420.6	213.5	353.1	1871.5	1489.3	1784.4
Save	Mean	3.9	4.6	3.8	135.6	96.5	144.1	1105.2	1153.3	1191.2
	Stdev	8.7	10.2	8	52.4	102.9	136.1	244.1	138.3	177.3
	Max	41.4	59.6	39.2	174.3	174.3	201.3	1935.9	1636.2	1851.5
Parakou	Mean	5.2	4.6	5	114.9	120.8	108.4	1164.7	1302.2	1279.7
	Stdev	11.3	9.6	12.5	51.1	94.6	105.2	224.3	149	168.6
	Max	64.5	58.2	67.4	188.9	219.7	200.1	1614.6	1733.2	1711.1
Kandi	Mean	3.4	4.2	3.9	90.3	115.3	99.2	1027.6	1320.1	1132.1
	Stdev	9.9	8.1	10.1	76.4	84.3	75.3	173.2	129.4	170.5
	Max	72.5	64.4	68.9	178.6	204.7	183.5	1471.6	1221.4	1345.1
Natitingou	Mean	4.1	4.9	3.8	99.9	120.3	108.2	1270.1	1320.5	1301.2
	Stdev	8.1	8.5	7.9	71.1	80.2	74.1	228.1	130.3	172.3
	Max	47.1	60.2	53.8	156.4	176.2	162.5	1898.4	1634.2	1802.5

Table 6: Statistics of daily, monthly and yearly rainfall depths (mm) for the investigated stations (OBS=observed data, Exp=exponential distribution, Gam=gamma distribution, Stdev=standard deviation and Max = maximum of the data).
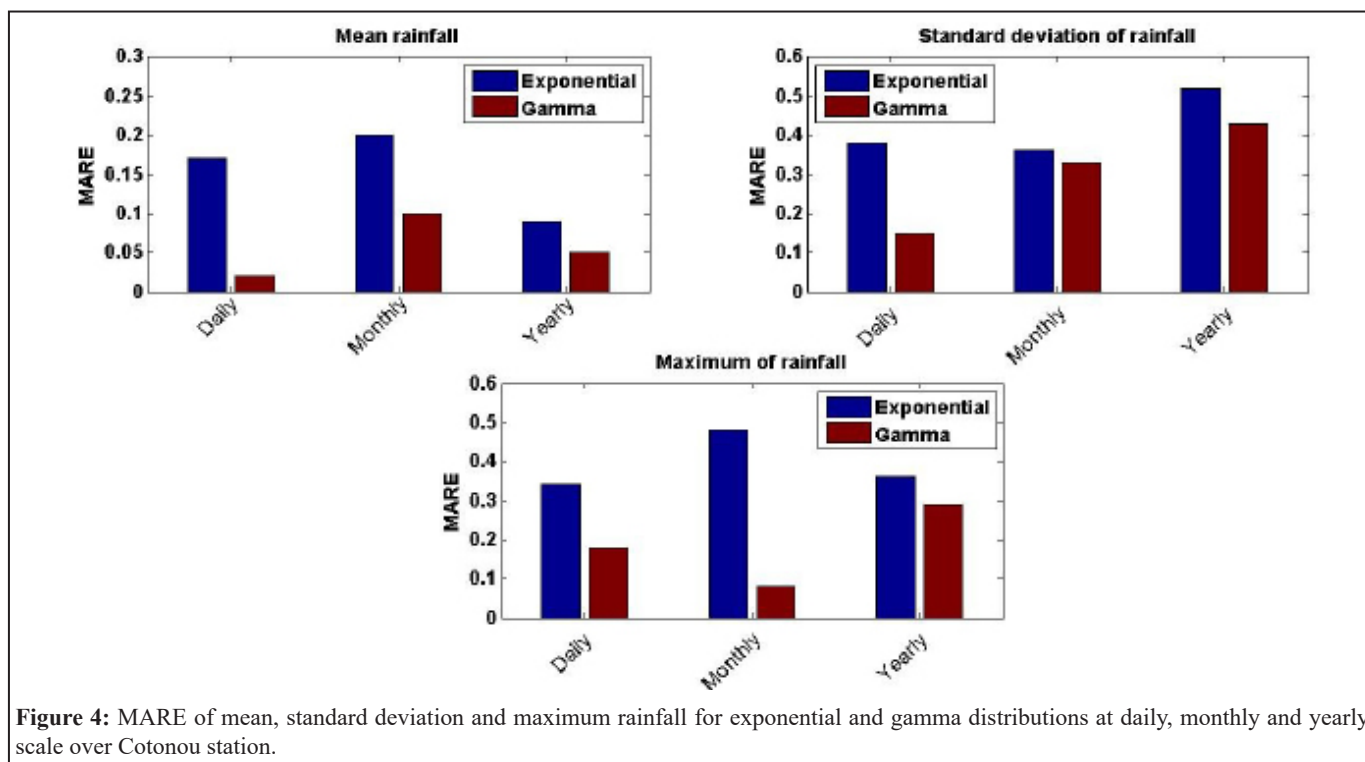
Figure 4: MARE of mean, standard deviation and maximum rainfall for exponential and gamma distributions at daily, monthly and yearly scale over Cotonou station.

exponential distribution in terms of generating rainfall data that have statistical properties comparable to those of existing records. Amount of rainfall is an important factor that impacts agriculture system. It governs the crop yields that determine the choice of the crops that can be grown.

Conclusion

The first and second-order Markov chain was used to analyse the wet and dry spells in Benin. The Markov chain modelling approach revealed that the probabilities of having two successive

dry days and the probabilities of having three successive dry days are highest among all other transitions probabilities. Moreover, transition probabilities show considerable variation throughout the rainy season. There was a systematic variation in the probability of a wet day following a dry day as one move northwards and a limited variation in the probability of a wet day following a wet day. Analysis of the characteristics of dry and wet spells duration reveals that dry spell duration fluctuates around the mean more than the wet spell duration. Then, the one-parameter exponential and the two-parameter gamma distributions were used to produce wet day rainfall amount. The results for the generation of daily rainfall showed that the two-parameter gamma distribution is consistently better than the one-parameter exponential distribution at simulating rainfall in the study area. We noticed that, for exponential distribution, the MARE varies from 0.31% to 0.38% for the mean rainfall, 0.28% to 0.35% for the standard deviation of rainfall and 0.41% to 0.48% for the maximum rainfall, whereas for the gamma distribution this performance criteria varies from 0.21% to 0.29% for the mean rainfall, 0.20% to 0.27% for the rainfall standard deviation and 0.30% to 0.40% for the maximum rainfall at daily, monthly and yearly scale over the six synoptic stations. This study has great important not only for data production purpose also for water resources management. Because information about the probability of the rainfall for future can be used to make decisions relating to water management, it can decrease risks originating from weather conditions uncertainties.

References

1. Selvi ST, Selvaraj RS (2011) Stochastic modelling of annual rainfall at Tamil Nadu. Universal Journal of Environmental Research and Technology 1: 566-570.

2. Selvaraj RS, Selvi ST (2010) Stochastic modelling of daily rainfall at Aduthurai. International Journal of Advanced Computer and Mathematical Sciences 1: 52-57.

3. Gabriel KR, Neumann J (1962) Markov chain model for daily rainfall occurrences at Tel Aviv. Quar J Roy Meteor Soc 88: 90-95.

4. Afouda A (1985) Contribution a la representation mathematique du fonctionnement hydrodynamique des bassins versants. These d'Etat no. 98, INP-Toulouse, France.

5. Afouda A (1989) Developpement de l'hydrologie theorique en Afrique de l'Ouest. In : Stout GE, Demissie (eds). Proc. Sahel Forum. IWRA, Urbana, Illinois, USA.

6. Afouda A (1990) On the stochastic structure of precipitation processes. In: Water for Life (Proc. Silver Jubilee Int. Centre of Hydrol. Dino Tonini, University of Padova, Italy. Pg no: 57-71.

7. Srikanthan R, McMahon TA (2001) Stochastic generation of annual, monthly and daily climate data Hydrology and Earth System Sciences 5: 653-670.

8. Jovanovic C, Dakkak AR, Cabric M, Brajkovic M (1974) Simulation of daily rainfall series using Markov chain models Math models in hydrology symposium. UNESCODOC 1: 110-120.

9. Sen Z (2015) Regional Wet and Dry spell analysis with heterogeneous probability occurrences. Journal of Hydrologic Engineering 20.

10. Obada E, Alamou AE, Chabi A, Zandagba J, Afouda A (2017) Trends and changes in Recent and Future Penman-Monteith Potential Evapotranspiration in Benin (West Africa). Hydrology 4: 38.

11. Ermert V, Brucher T (2008) The climate of Benin (1961-1990). In: Judex M, Thamm HP (eds). Impetus Atlas Benin. Research Results 2000-2007. 3rdedn, Department of Geography, University of Bonn, Germany.

12. Giertz S, Hiepe C (2008) Soil map of Benin. In: Judex M, Thamm HP (eds). Impetus Atlas Benin. Research Results 2000-2007. 3rdedn, Department of Geography, University of Bonn, Germany.

13. De Michele C, Bernardara P (2005) Spectral analysis and modeling of space-time rainfall fields. Atmospheric Research1-4: 124-136.

14. Katz RW (1977) An application of chain-dependent processes to meteorology. Journal of Applied Probability 14: 598-603.

15. Katz RW(1977) Precipitation as a chain dependent process. Journal of applied Meteorology 16: 671-676.

16. Swift LW, Schreuder HT (1981) Fitting daily precipitation amounts using the SB distribution. Mon Wea Rev 109: 2535-2541.

17. Wilks DS (1998) Multisite generalization of a daily stochastic precipitation generation model. J Hydrol 210: 178-191.

18. Wilks DS (1999) Interannual variability and extreme-value characteristics of several stochastic daily precipitation models. Agric For Meteor 93: 153-169.

19. Gates P, Tong H (1976) On Markov chain modelling to some weather data. J Appl Meteorol 15: 1145-1151.

20. Buishand TA (1978) Some remarks on the use of daily rainfall models. J Hydrol 36: 295-308.

21. Coe R, Stern RD (1982) Fitting models to daily rainfall data. J Appl Meteor 21: 1024-1031.

22. Katz RW, Parlange MB (1998) Overdispersion phenomenon in stochastic modeling of precipitation. J Climate 11: 591-601.

23. Madden RA, Shea DJ, Katz RW, Kidson JW (1999) The potential long-range predictability of precipitation over New Zealand. Int J Climatol 19: 405-421.

24. Gabriel KR, Neumann J (1957) Distribution of weather cycles by lengths. Quart J R met Soc 83: 375-380.

25. Gabriel KR, Neumann JA (1961) Markov chain for daily rainfall occurrence at Tel Aviv. Quart J R met Soc 88: 90-95.

26. Chen J, Brissette FP, Leconte R (2012) WeaGETS - a Matlab-based daily scale weather generator for generating precipitation and temperature. Procedia Environmental Sciences 13: 2222-2235.

27. Afouda A, Adisso P, Etude stochastique des structures de sequences seches au Benin (1997) Sustainability of Water Resources under Increasing Uncertainty (Proceedings of the Rabat Symposium S1), IAHS Publ. Pg no: 240.

28. Wagoussi OT (2012) Modelisation des precipitations au Benin par une approche Markovienne. Memoire de Master Professionnel en Hydrologie Pg: 115.

29. Jimoh OD, Webster P (1999) Stochastic modelling daily rainfall in Nigeria, intra-annual variation of model parameters. J Hydrol 222: 1-17.